|  |  |
| --- | --- |
| Name:  **Year 12 Methods**  **Test 4 2019 Calculator Free:**Discrete Random Variables, Bernoulli and Binomial Distributions |  |

**Reading Time:** 1 minutes **Working Time:** 20 minutes

**Materials:** Formula Sheet **20** **Marks**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Question 1 (6 marks)

The table below shows the probability distribution for a random variable .

It is known that and .

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |

(a) Determine the values of the constants and . (4 marks)

(b) Determine

(i) . (1 mark)

(ii) . (1 mark)

Question 2 (7 marks)

A calculator program will generate a single random integer , where . The program is run once, and the random variable is the number of fours or fives obtained.

(a) Explain why is a Bernoulli random variable. (1 mark)

(b) Determine . (1 mark)

(c) Determine the mean and standard deviation of . (2 marks)

The random variable is the number of fours or fives obtained in three consecutive runs of the program.

(d) Determine . (3 marks)

Question 3 (7 marks)

A farmer keeps a brood of hens that can each lay up to one egg per day. On any given day, the probability that a hen lays an egg is independent with a constant value of .

The discrete random variable is the number of eggs laid by the brood in one day and has a mean of 3 and standard deviation of .

(a) State the name given to this type of probability distribution and briefly explain why it is discrete.

(2 marks)

(b) Determine the value of and the value of . (3 marks)

(c) Determine the mean and variance of the distribution , where . (2 marks)

|  |  |
| --- | --- |
| Name:  **Year 12 Methods**  **Test 4 2019 Calculator Assumed:**Discrete Random Variables, Bernoulli and Binomial Distributions |  |

**Reading Time:** 1 minutes **Working Time:** 36 minutes

**Materials:** 1 page of notes and CAS Calculator **36** **Marks**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Question 4 (7 marks)

(a) Four random variables , , and are defined below. State, with reasons, whether the distribution of the random variable is Bernoulli, binomial, uniform or none of these.

*The dice referred to is a cube with faces numbered with the integers 1, 2, 3, 4, 5 and 6.*

(4 marks)

(i) is the number of throws of a dice until a six is scored.

(ii) is the score when a dice is thrown.

(iii) is the number of odd numbers showing when a dice is thrown.

(iv) is the total of the scores when two dice are thrown.

(b) Pegs produced by a manufacturer are known to be defective with probability , independently of each other. The pegs are sold in bags of for $4.95. The random variable is the number of faulty pegs in a bag.

If and , determine and . (3 marks)

Question 5 (10 marks)

A slot machine is programmed to operate at random, making various payouts after patrons pay $2 and press a start button. The random variable is the amount of the payout, in dollars, in one play of the machine. Each payout can be assumed to be independent of other payouts.

The probability, , that the machine makes a certain payout, , is shown in the table below.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Payout ($) | 0 | 1 | 2 | 5 | 10 | 20 | 50 | 100 |
| Probability | 0.25 | 0.45 | 0.2125 | 0.0625 | 0.0125 | 0.005 | 0.005 | 0.0025 |

(a) Determine the probability that

(i) in one play of the machine, a payout of more than $1 is made. (1 mark)

(ii) in ten plays of the machine, it makes a payout of $5 no more than once. (2 marks)

(iii) in five plays of the machine, the second payout of $1 occurs on the fifth play. (3 marks)

(b) Calculate the mean and standard deviation of . (2 marks)

(c) In the long run, what percentage of the patron's money is returned to them? (2 marks)

Question 6 (10 marks)

Let the random variable be the number of vowels in a random selection of four letters from those in the word LOGARITHM, with no letter to be chosen more than once.

(a) Complete the probability distribution of below. (1 mark)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |

(b) Show how the probability for was calculated. (2 marks)

(c) Determine . (2 marks)

Let event occur when no vowels are chosen in random selection of four letters from those in the word LOGARITHM.

(d) State . (1 mark)

(e) Let be a Bernoulli random variable with parameter . Determine the mean and standard deviation of . (2 marks)

(f) Determine the probability that occurs no more than twice in ten random selections of four letters from those in the word LOGARITHM. (2 marks)

Question 7 (9 marks)

75% of the avocados produced by a farm are known to be first grade, the rest being second grade. Trays of 24 avocados are filled at random in a packing shed and sent to market.

Let the random variable be the number of first grade avocados in a single tray.

(a) Explain why is a discrete random variable, and identify its probability distribution.

(2 marks)

(b) Calculate the mean and standard deviation of . (2 marks)

(c) Determine the probability that a randomly chosen tray contains

(i) 18 first grade avocados. (1 mark)

(ii) more than 15 but less than 20 first grade avocados. (2 marks)

(d) In a random sample of 1000 trays, how many trays are likely to have fewer first grade than second grade avocados. (2 marks)